

# International Journal of Engineering Sciences & Research Technology

(A Peer Reviewed Online Journal)  
Impact Factor: 5.164



**Chief Editor**  
Dr. J.B. Helonde

**Executive Editor**  
Mr. Somil Mayur Shah

## ABSTRACT

We have suggested an improved ratio type log estimator for population variance by using coefficient of kurtosis and median of an auxiliary variable  $x$ . The properties of proposed estimator have been derived up to first order of Taylor's series expansion. The efficiency conditions derived theoretically under which the proposed estimator performs better than existing estimators. The proposed estimator as illustrated by the empirical studies using real populations performs better than the existing estimators i.e. it has the smallest Mean Squared Error and the highest Percentage Relative Efficiency.

**KEYWORDS:** Ratio Type Variance Estimator, Auxiliary Variable, Kurtosis, Median, Bias, Mean Squared Error (MSE), Percentage Relative Efficiency (PRE), Simple Random Sampling.

## 1. INTRODUCTION

Consider a finite population of size  $N$  identifiable and non-overlapping units. From this population size, a sample of size  $n$  is drawn by simple random sampling. Let, the variable under study is denoted by  $y$  and the variable which contains the auxiliary information about the study variable is denoted by  $x$ . The population mean of study and auxiliary variable is denoted by  $\bar{Y}$  and  $\bar{X}$ . Further, the population and sample variances are given as under

$$S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})^2, S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2 \text{ and}$$

$$s_y^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{y})^2, s_x^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2$$

In general, we define the following parameters

$$\mu_{rs} = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{y})^r (x_i - \bar{x})^s, \lambda_{22} = \frac{\mu_{22}}{\mu_{02} \mu_{20}}, C_y \text{ and } C_x \text{ be the coefficient of variation of the study and auxiliary variable.}$$

$$K_x = \lambda_{04} = \frac{\mu_{04}}{\mu_{02}^2} \text{ and } K_y = \lambda_{40} = \frac{\mu_{40}}{\mu_{20}^2} \text{ be the coefficient of kurtosis of the study and auxiliary variable.}$$

$$\rho_{xy} = \frac{s_{xy}}{s_y s_x} \text{ be the coefficient of correlation between } y \text{ and } x.$$

$M_x$  be the population median of auxiliary variable.

Many authors have come up with more precise estimators by employing prior knowledge of certain population parameter(s). [2] for example attempted use of the coefficient of variation of study variable but prove inadequate for in practice, this parameter is unknown. Motivated by [2] work, [3] [4] and [5] used the known coefficient of variation but now that of the auxiliary variable for estimating population mean of study variable. Reasoning along the same path [6] used the prior value of coefficient of kurtosis of an auxiliary variable in estimating the population variance of the study variable  $y$ . Kurtosis in most cases is not reported or used in many research articles, in spite of the fact that fundamentally speaking every statistical package provides a measure of kurtosis. This may be attributed to the likelihood that kurtosis is not well understood or its importance in various aspects of statistical analysis has not been explored fully. Kurtosis can simply be expressed as

$$K = \frac{E(x - \mu)^4}{(E(x - \mu)^2)^2}$$

where E denotes the expectation,  $\mu$  is the mean. Moreover, one of the measure of central tendency which we use in this article is Median. Median is the middle term of the series of the distribution under study when the values are arranged in ascending or descending order. In spite of many applications of median, the most important advantage of median is that it is less affected by the outliers and skewed data, thus is preferred to the mean especially when the distribution is not symmetrical. We can therefore utilize the median and the coefficient of kurtosis of the auxiliary variable to derive a more precise ratio type log estimator for population variance.

**2. ESTIMATORS AVAILABLE IN LITERATURE**

In this section we have reviewed some finite population variance estimators existing in literature which will help in the construction and development of the proposed estimator. Notably, when auxiliary information is not available the usual unbiased estimator to the population variance is

$$t_1 = s_y^2 \tag{1}$$

The bias and MSE of  $t_1$  is

$$Bias(t_1) = \frac{1-f}{n} S_y^2 \left\{ (K_x - 1) \Psi_1 \left( \Psi_1 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} = 0 \tag{2}$$

$$MSE(t_1) = V(t_1) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_1 \left( \Psi_1 - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{3}$$

$$= \frac{1-f}{n} S_y^4 (K_y - 1); \text{ where } \Psi_1 = 0$$

Population variance, estimation using auxiliary information was considered by [7], and proposed ratio type population variance estimator, given by

$$t_2 = \frac{S_x^2}{s_x^2} S_y^2 \tag{4}$$

The bias and Mean Squared Error of Isaki's estimator,

$$Bias(t_2) = \frac{1-f}{n} S_y^2 \left\{ (K_x - 1) \Psi_2 \left( \Psi_2 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} = \frac{1-f}{n} S_y^2 \left\{ (K_x - 1) - (\lambda_{22} - 1) \right\}$$

$$MSE(t_2) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_2 \left( \Psi_2 - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{5}$$

$$= \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) - 2(\lambda_{22} - 1) \right\}; \text{ where } \Psi_2 = 1$$

[6] initiated the use of coefficient of kurtosis in estimating population variance of a study variable y. Later, the coefficient of kurtosis was used by [3] [5] [8] in the estimating the population mean. [9] using the known information on both  $S_x^2$  and  $k_x$  suggested modified ratio type population variance estimator for  $S_y^2$  as

$$t_3 = \left[ \frac{S_x^2 + k_x}{s_x^2 + k_x} \right] S_y^2 \tag{6}$$

The estimator, bias and MSE obtained as

$$Bias(t_3) = \frac{1-f}{n} S_y^2 \left\{ (K_x - 1) \Psi_3 \left( \Psi_3 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \tag{7}$$

$$MSE(t_2) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_3 \left( \Psi_3 - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{8}$$

where  $\Psi_3 = \left[ \frac{S_x^2}{S_x^2 + k_x} \right]$

[10] suggested four modified ratio type variance estimators using known values of  $C_x$  and  $k_x$ ,

$$t_4 = \left[ \frac{S_x^2 - C_x}{s_x^2 - C_x} \right] S_y^2 \tag{9}$$

$$t_5 = \left[ \frac{S_x^2 - k_x}{s_x^2 - k_x} \right] S_y^2 \tag{10}$$

$$t_6 = \left[ \frac{S_x^2 k_x - C_x}{s_x^2 k_x - C_x} \right] S_y^2 \tag{11}$$

$$t_7 = \left[ \frac{S_x^2 C_x - k_x}{s_x^2 C_x - k_x} \right] S_y^2 \tag{12}$$



The biases and MSE of their estimators,

$$Bias(t_4) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_4 \left( \Psi_4 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \quad (13)$$

$$MSE(t_4) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_4 \left( \Psi_4 - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \quad (14)$$

$$Bias(t_5) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_5 \left( \Psi_5 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \quad (15)$$

$$MSE(t_5) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_5 \left( \Psi_5 - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \quad (16)$$

$$Bias(t_6) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_6 \left( \Psi_6 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \quad (17)$$

$$MSE(t_6) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_6 \left( \Psi_6 - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \quad (18)$$

$$Bias(t_7) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_7 \left( \Psi_7 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \quad (19)$$

$$MSE(t_7) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_7 \left( \Psi_7 - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \quad (20)$$

Where

$$\Psi_4 = \left[ \frac{S_x^2}{S_x^2 - C_x} \right], \Psi_5 = \left[ \frac{S_x^2}{S_x^2 - K_x} \right], \Psi_6 = \left[ \frac{S_x^2 K_x}{S_x^2 K_x - C_x} \right], \Psi_7 = \left[ \frac{S_x^2 C_x}{S_x^2 C_x - K_x} \right]$$

[11] utilizing population median  $M_x$  came up with a modified ratio type population variance estimator as

$$t_8 = \left[ \frac{S_x^2 - M_x}{S_x^2 - M_x} \right] S_y^2 \quad (21)$$

The bias and MSE of their estimator

$$Bias(t_8) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_8 \left( \Psi_8 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \quad (22)$$

$$MSE(t_8) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_8 \left( \Psi_8 - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \quad (23)$$

Where  $\Psi_8 = \left[ \frac{S_x^2}{S_x^2 + M_x} \right]$

[12] using the known quartiles (upper and lower quartile  $Q_3$  and  $Q_1$  respectively) of the auxiliary variable  $x$  suggested

$$t_9 = \left[ \frac{S_x^2 + Q_1}{S_x^2 + Q_1} \right] S_y^2 \quad (24)$$

$$t_{10} = \left[ \frac{S_x^2 + Q_3}{S_x^2 + Q_3} \right] S_y^2 \quad (25)$$

The biases and MSE of their estimators  $t_9$  and  $t_{10}$  as follows

$$Bias(t_9) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_9 \left( \Psi_9 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \quad (26)$$

$$MSE(t_9) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_9 \left( \Psi_9 - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \quad (27)$$

$$Bias(t_{10}) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_{10} \left( \Psi_{10} - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \quad (28)$$

$$MSE(t_{10}) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_{10} \left( \Psi_{10} - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \quad (29)$$

Where  $\Psi_9 = \left[ \frac{S_x^2}{S_x^2 + Q_1} \right]$  and  $\Psi_{10} = \left[ \frac{S_x^2}{S_x^2 + Q_3} \right]$

Motivated by [10] and [11] [13] considered the estimation of finite population variance using known coefficient of variation and median of an auxiliary variable, proposed an estimator.

$$t_{11} = \left[ \frac{C_x S_x^2 + M_x}{C_x S_x^2 + M_x} \right] S_y^2 \quad (30)$$

The bias and MSE obtained to be,

$$Bias(t_{11}) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_{11} \left( \Psi_{11} - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \quad (31)$$

$$MSE(t_{11}) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_{11} \left( \Psi_{11} - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \quad (32)$$

where  $\Psi_{11} = \left[ \frac{C_x S_x^2}{C_x S_x^2 + M_x} \right]$

Motivated by [23] considered the estimation of finite population variance using known kurtosis and median of an auxiliary variable.

$$t_{12} = S_y^2 \left\{ \frac{K_x S_x^2 + M_x^2}{K_x S_x^2 + M_x^2} \right\} \quad (33)$$

The bias and MSE obtained to be,

$$Bias(t_{12}) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_{12} \left( \Psi_{12} - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \quad (34)$$

$$MSE(t_{12}) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_{12} \left( \Psi_{12} - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \quad (35)$$

where  $\Psi_{12} = \left[ \frac{K_x S_x^2}{C_x S_x^2 + M_x^2} \right]$

### 3. PROPOSED ESTIMATOR

Motivated by the works of [23] [24] and [25] in the improvement of the performance of the population variance estimator of the study variable using known population parameters of an auxiliary variable. We introduce the following improved ratio type log estimator for population variance using a known value of population coefficient of kurtosis  $k_x$  and median  $M_x$  of an auxiliary variable.

$$t_{13} = w_1 S_y^2 \left[ 1 + \log \left\{ \frac{K_x S_x^2 + M_x^2}{K_x S_x^2 + M_x^2} \right\} \right]^a \quad (36)$$

To calculate the bias and the MSE of  $t_{13}$ ;

We let,

$$\epsilon_0 = \frac{(S_y^2 - s_y^2)}{S_y^2}, \epsilon_1 = \frac{(S_x^2 - s_x^2)}{S_x^2}$$

$$E(\epsilon_0) = E(\epsilon_1) = 0, E(\epsilon_0^2) = \frac{1-f}{n} (\lambda_{40} - 1), E(\epsilon_1^2) = \frac{1-f}{n} (\lambda_{04} - 1),$$

$$E(\epsilon_0 \epsilon_1) = \frac{1-f}{n} (\lambda_{22} - 1)$$

Now expressing  $t_{13}$  in terms of epsilons  $\epsilon$  we have,

$$\begin{aligned} t_{13} &= w_1 S_y^2 (1 + \epsilon_0) \left[ 1 + \log \left\{ \frac{K_x S_x^2 + M_x^2}{K_x S_x^2 (1 + \epsilon_1) + M_x^2} \right\} \right]^a \\ &= w_1 S_y^2 (1 + \epsilon_0) \left[ 1 + \log \left\{ 1 + \frac{K_x S_x^2 \epsilon_1}{(K_x S_x^2 + M_x^2)} \right\}^{-1} \right]^a \\ &= w_1 S_y^2 (1 + \epsilon_0) [1 + \log\{1 + \eta_{13} \epsilon_1\}^{-1}]^a \end{aligned} \quad (37)$$

where  $\eta_{13} = \left[ \frac{K_x S_x^2}{K_x S_x^2 + M_x^2} \right]$ , we assume that  $|\eta_{13} \epsilon_1| < 1$ , so that  $\{1 + \eta_{13} \epsilon_1\}^{-1}$  is expandable.

Expanding the right hand side of (37) and multiplying out we have

$$\begin{aligned} &= w_1 S_y^2 (1 + \epsilon_0) [1 + \log\{1 - \eta_{13} \epsilon_1 + (\eta_{13} \epsilon_1)^2\}]^a \\ t_{13} - S_y^2 &= (w_1 - 1) S_y^2 + w_1 S_y^2 \left[ \epsilon_0 - a \eta_{13} \epsilon_0 \epsilon_1 - a \eta_{13} \epsilon_1 + \frac{a^2}{2} \eta_{13}^2 \epsilon_1^2 \right] \end{aligned} \quad (38)$$

Taking expectations on both the sides, we get

$$E(t_{13} - S_y^2) = (w_1 - 1) S_y^2 + w_1 S_y^2 \left[ -a \eta_{13} E(\epsilon_0 \epsilon_1) + \frac{a^2}{2} \eta_{13}^2 E(\epsilon_1^2) \right]$$

$$Bias(t_{13}) = S_y^2 \left[ (w_1 - 1) + w_1 \left( \frac{1-f}{n} \right) \left\{ (K_x - 1) a \eta_{13} \left( \frac{a}{2} \eta_{13} - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \right]$$

(39)

Squaring on both the sides of equation (38), we get MSE

$$MSE(t_{13}) = S_y^4 + S_y^4 w_1^2 \left[ 1 + \left( \frac{1-f}{n} \right) \left\{ (K_y - 1) + 2(K_x - 1) a \eta_{13} \left( a \eta_{13} - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \right]$$



$$-2 w_1 S_y^4 \left[ 1 + \frac{1-f}{n} \left\{ (K_x - 1) a \eta_{13} \left( \frac{a}{2} \eta_{13} - \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \right] \tag{40}$$

On differentiating  $MSE(t_{13})$  with respect to  $w_1$ , we get

$$w_{1 \text{ opt}} = \left[ \frac{B}{A} \right]$$

Where  $A = \left[ 1 + \left( \frac{1-f}{n} \right) \left\{ (K_y - 1) + 2(K_x - 1) a \eta_{13} \left( a \eta_{13} - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \right]$  and

$$B = \left[ 1 + \frac{1-f}{n} \left\{ (K_x - 1) a \eta_{13} \left( \frac{a}{2} \eta_{13} - \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \right] \tag{41}$$

$$[MSE(t_{13})]_{min} = S_y^4 \left\{ 1 - \frac{B^2}{A} \right\} \tag{42}$$

is the desired optimum mean squared estimator for proposed estimator.

#### 4. THEORETICAL COMPARISONS

The theoretical conditions under which the proposed modified ratio type estimators  $t_{12}$  is more efficient than the other existing estimator  $t_j, j=1, 2, \dots, 11, 12$  from MSE of  $t_j, j=1, 2, \dots, 11, 12$ ; given to the first degree of approximation in general as

$$MSE(t_j) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \psi_j \left( \psi_j - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{43}$$

Using Equations (40) and (43), we have  $MSE(t_{13}) < MSE(t_j)$ , if

$$a \eta_{13} \left( a \eta_{13} - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) < \psi_j \left( \psi_j - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \tag{44}$$

#### Empirical Studies

Using the data from Population I (Source:[21], 228]) and Population II (source: [22]). We perform the working of proposed and existing estimators to this real data set and the data statistics are given below:

##### Population I

X = Fixed capital, Y = output of 80 factories,  $N = 80, n = 20, \bar{X} = 11.265, \bar{Y} = 51.826, S_x^2 = 71.504, S_y^2 = 336.979, S_{xy} = 146.068, \lambda_{04} = K_x = 2.866, \lambda_{40} = K_y = 2.267, \lambda_{22} = 2.221, \rho_{xy} = 0.941, C_y = 0.354, C_x = 0.751, M_x = 10.300, Q_1 = 5.150, Q_3 = 16.975$

##### Population II

X = acreage under wheat crop in 1973, Y = acreage under wheat crop in 1974,  $N = 70, n = 25, \bar{X} = 175.2671, \bar{Y} = 96.700, S_x^2 = 19840.7508, S_y^2 = 3686.1898, \lambda_{04} = K_x = 7.0952, \lambda_{40} = K_y = 4.7596, \lambda_{22} = 4.6038, \rho_{xy} = 0.7293, C_y = 0.6254, C_x = 0.8037, M_x = 72.4375, Q_1 = 80.1500, Q_3 = 225.0250$

Using the above summary values we have the results in Table 1 below. From the table, Mean Squared Errors it is clear that our proposed ratio type log estimator  $t_{13}$  for population variance has the least Mean Squared Error (MSE). The efficiency of our proposed estimator  $t_{13}$  is examined numerically by its Percentage Relative Efficiency (PRE(s)) in comparison with those of existing

Table 1. Bias and Mean Squared Errors (MSE)

Estimator	Population I		Population II	
	Bias	MSE	Bias	MSE
$t_1$	0	5395.289	0	1313625.261
$t_2$	8.151	3276.421	236.154	924946.481
$t_3$	6.956	2740.349	235.656	924324.375
$t_4$	8.512	3006.373	236.187	925017.011
$t_5$	9.518	3186.399	236.445	925569.577

$t_6$	8.279	2965.067	236.159	924956.421
$t_7$	10.002	3275.722	236.517	925721.916
$t_8$	4.530	2377.418	233.201	918641.426
$t_9$	6.126	2609.91	232.889	917976.121
$t_{10}$	2.934	2181.488	227.099	905689.896
$t_{11}$	3.656	2314.033	232.485	917116.922
$t_{12}$	0.708	1993.270	207.653	865134.030
$t_{13}$	<b>-3.306</b>	<b>8.577</b>	<b>-117.995</b>	<b>6252.648</b>

estimators using real populations from [[21], p.228] and [22].

We have computed the PRE(s) of the estimators  $t_j, j=1, 2, \dots, 11,12$  using the formulae

$$PRE = \frac{MSE(s_y^2)}{MSE(t_j)} \times 100$$

$$PRE = \frac{\frac{1-f}{n} S_y^4 (K_y-1)}{\frac{1-f}{n} S_y^4 \left\{ (K_y-1) + (K_x-1) \psi_j \left( \psi_j - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\}} \times 100$$

$$= \frac{(K_y - 1)}{\left\{ (K_y - 1) + (K_x - 1) \psi_j \left( \psi_j - 2 \left( \frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\}}$$

Then PRE for our proposed estimator is subsequently,

$$PRE = \frac{MSE(s_y^2)}{MSE(t_{13})} \times 100$$

$$= \frac{\frac{1-f}{n} S_y^4 (K_y-1)}{S_y^4 \left\{ 1 - \frac{B^2}{A} \right\}} \times 100$$

Using formula (40) and (43) we compute the Percent Relative Efficiencies and tabulate the results in Table 2.

Percentage Relative efficiency being a robust statistical tool that is used to ascertain the efficiency of suggested estimator over conventional estimator. From the findings summarized in the table above it is clear that our proposed estimator  $t_{13}$  performed best, that is it has the highest PRE among all the other estimators. This therefore implies that we can apply our proposed estimator to appropriate practical situations and obtain better and more efficient results than the traditional and other existing population variance estimators.

**Table 2. Percent Relative Efficiencies (PRE)**

Estimator	PRE of Population I	PRE of Population II
$t_1$	100	100
$t_2$	164.67	142.02
$t_3$	196.88	142.12
$t_4$	179.46	142.01
$t_5$	169.32	141.93
$t_6$	181.96	142.02
$t_7$	164.71	141.90
$t_8$	226.94	143.00
$t_9$	206.72	143.10
$t_{10}$	247.32	145.04
$t_{11}$	233.16	143.23
$t_{12}$	270.68	151.84

$t_{13}$	2358.63	265.51
----------	---------	--------

## 5. CONCLUSION

In this article, we have proposed an improved log type ratio estimator for population variance estimator using known coefficient of kurtosis and the median of the auxiliary variable  $x$ . We have analyzed the performance of our proposed estimator against the usual unbiased variance estimator and existing estimators using two natural populations by comparing their PRE(s). Based on the results of our studies, it is proved that our proposed estimator works better than the other existing estimators having the highest Percentage Relative Efficiency. Hence can be applied to practical applications, where knowledge of population parameters of auxiliary variable is available. We also recommend that our proposed estimator can be further improved by extending the number of Taylor's series terms to be more than two.

## REFERENCES

- [1] Cochran, W. (1940) The Estimation of the Yields of Cereal Experiments by Sampling for the Ratio of Grain to Total Produce. The Journal of Agricultural Science , 30, 262-275. <https://doi.org/10.1017/S0021859600048012>
- [2] Searls, D.T. (1964) The Utilization of a Known Coefficient of Variation in the Estimation Procedure. Journal of the American Statistical Association , 59, 1225-1226. <https://doi.org/10.1080/01621459.1964.10480765>
- [3] Sen, A.R. (1978) Estimation of the Population Mean When the Coefficient of variation Is Known. Communications in Statistics — Theory and Methods , 7, 657-672. <https://doi.org/10.1080/03610927808827656>
- [4] Sisodia, B. and Dwivedi, V. (1981) Modified Ratio Estimator Using Coefficient of Variation of Auxiliary Variable. Journal - Indian Society of Agricultural Statistics , 33, 13-18.
- [5] Upadhyaya, L.N. and Singh, H.P. (1984) On the Estimation of the Population Mean With Known Coefficient of Variation. Biometrical Journal , 26, 915-922. <https://doi.org/10.1002/bimj.4710260814>
- [6] Hirano, K., Pandey, B.N. and Singh, J. (1973) On the Utilization of a Known Coefficient of Kurtosis in the Estimation Procedure of Variance. Annals of the Institute of Statistical Mathematics , 25, 51-55. <https://doi.org/10.1007/BF02479358>
- [7] Isaki, C.T. (1983) Variance Estimation Using Auxiliary Information. Journal of the American Statistical Association , 78, 117-123. <https://doi.org/10.1080/01621459.1983.10477939>
- [8] Searls, D.T. and Intarapanich, P. (1990) A Note on an Estimator for the Variance That Utilizes The kurtosis. The American Statistician , 44, 295-296.
- [9] Upadhyaya, L. and Singh, H. (1999) An Estimator for Population Variance That Utilizes the Kurtosis of an Auxiliary Variable in Sample Surveys. Vikram Math ematical Journal , 19, 14-17.
- [10] Kadilar, C. and Cingi, H. (2006) Ratio Estimators for the Population Variance in Simple and Stratified Random Sampling. Applied Mathematics and Computation , 173, 1047-1059. <https://doi.org/10.1016/j.amc.2005.04.032>
- [11] Subramani, J. and Kumarapandiyan, G. (2012) Variance Estimation Using Median of the Auxiliary Variable. International Journal of Probability and Statistics , 1, 6-40. <https://doi.org/10.5923/j.ijps.20120103.02>
- [12] Subramani, J. and Kumarapandiyan, G. (2012) Variance Estimation Using Quartiles and Their Functions of an Auxiliary Variable. International Journal of Statistics and Applications , 2, 67-72. <https://doi.org/10.5923/j.statistics.20120205.04>
- [13] Subramani, J. and Kumarapandiyan, G. (2013) Estimation of Variance Using Known Coefficient of Variation and Median of an Auxiliary Variable. Journal of Modern Applied Statistical Methods , 12, 11. <https://doi.org/10.22237/jmasm/1367381400>
- [14] Khan, M. and Shabbir, J. (2013) A Ratio Type Estimator for the Estimation of Population Variance Using Quartiles of an Auxiliary Variable. Journal of Statistics Applications and Probability , 2, 157-162. <https://doi.org/10.12785/jsap/020314>
- [15] Singh, H.P., Tailor, R., Tailor, R. and Kakran, M. (2004) An Improved Estimator of Population Mean Using Power Transformation. Journal of the Indian Society of Agricultural Statistics , 58, 223-230.





- [16] Yadav, S.K., Misra, S. and Mishra, S. (2016) Efficient Estimator for Population Variance Using Auxiliary Variable. *American Journal of Operational Research* , 6, 9-15.
- [17] Sukhatme, P. (1944) Moments and Product Moments of Moment-Statistics for Samples of the Finite and Infinite Populations. *Sankhy a: The Indian Journal of Statistics* , 6, 363-382.
- [18] Sukhatme, P. and Sukhatme, B. (1970) *Sampling Theory of Surveys with Applications*. Asia Publishing House, Bombay.
- [19] Srivastava, S.K. and Jhajj, H.S. (1981) A Class of Estimators of the Population Mean in Survey Sampling Using Auxiliary Information. *Biometrika* , 68, 341-343. <https://doi.org/10.1093/biomet/68.1.341>
- [20] Tracy, D.S. (1984) Moments of Sample Moments. *Communications in Statistics - Theory and Methods* , 13, 553-562. <https://doi.org/10.1080/03610928408828700>
- [21] Murthy, M.N. (1967) *Sampling Theory and Methods*. Statistical Publishing Society, Barrackpore.
- [22] Daroga, S. and Chaudhary, F. (1986) *Theory and Analysis of Sample Survey Designs*. Wiley Eastern Limited, New York.
- [23] T. K. Milton et al. (2017) Estimation of Population Variance Using the Coefficient of Kurtosis and Median of an Auxiliary Variable under Simple Random Sampling, *Open Journal of Statistics*, 7, 944-955.
- [24] Kumari et al. (2019). Optimal Two Parameter Logarithmic Estimators for Estimating the Population Variance, *Global Journal of Pure and Applied Statistics*, 15(5), 527-535.
- [25] Kumari et al. (2018). A Class of Double Sampling Log Type Estimators for Population Variance Using Two Auxiliary Variable, *International Journal of Applied Engineering Research*, 13, 11151-1115.

